Publication abstracts for Stefan Forcey

- New Hopf structures on binary trees. (with A. Lauve & F. Sottile)
  to appear, DMTCS Proceedings, 21st Annual Conference on Formal Power Series and Algebraic Combinatorics. (Hagenberg, Austria, 2009).

Arxiv: http://arxiv.org/abs/0908.3709
Preprint: http://faculty.tnstate.edu/sforcey/msym_fpsac.pdf

Abstract: The multiplihedra $\mathcal{M}_\bullet = (\mathcal{M}_n)_{n\geq 1}$ form a family of polytopes originating in the study of higher categories and homotopy theory. While the multiplihedra may be unfamiliar to the algebraic combinatorics community, it is nestled between two families of polytopes that certainly are not: the permutahedra $\mathfrak{S}_\bullet$ and associahedra $\mathcal{Y}_\bullet$. The maps $\mathfrak{S}_\bullet \twoheadrightarrow \mathcal{M}_\bullet \twoheadrightarrow \mathcal{Y}_\bullet$ reveal several new Hopf structures on tree-like objects nestled between the Hopf algebras $\mathfrak{S}Sym$ and $\mathcal{Y}Sym$. We begin their study here, showing that $\mathcal{M}Sym$ is a module over $\mathfrak{S}Sym$ and a Hopf module over $\mathcal{Y}Sym$. Rich structural information about $\mathcal{M}Sym$ is uncovered via a change of basis using Möbius inversion in posets built on the 1-skeleta of $\mathcal{M}_\bullet$. Our analysis uses the notion of an interval retract, which should have independent interest in poset combinatorics. It also reveals new families of polytopes, and even a new factorization of a known projection from the associahedra to hypercubes.

- Marked tubes and the graph multiplihedron. (with S.L. Devadoss)
  Algebraic and Geometric Topology, 8(4) 20812108, 2008.

Arxiv: http://arxiv.org/abs/0807.4159
Journal: http://www.msp.warwick.ac.uk/agt/2008/08-04/p074.xhtml

Abstract: Given a graph G, we construct a convex polytope whose face poset is based on the marked subgraphs of G. Dubbed the *graph multiplihedra*, we provide a realization using integer coordinates. Not only does this provide a natural generalization of the multiphihedron, but features of this polytope appear in works related to quilted disks, bordered Riemann surfaces, and operadic structures. Certain examples of graph multiplihedra are related to Minkowski sums of simplices and cubes and others to the permutohedron.

- Convex Hull Realizations of the Multiplihedra
  Topology and its Applications, 156, 326-347, 2008.

Arxiv: http://arxiv.org/abs/0706.3226
Journal: http://dx.doi.org/10.1016/j.topol.2008.07.010

Abstract: We present a simple algorithm for determining the extremal points in Euclidean space whose convex hull is the n^{th} polytope in the sequence known as the multiplihedra. This answers the open question of whether the multiplihedra could be realized as convex polytopes. We use this realization to unite the approach to A_n-maps of Iwase and Mimura to that of Boardman and Vogt. We include a review of the appearance of the n^{th} multiplihedron for various n in the studies of higher homotopy commutativity, (weak) n-categories, A_∞-categories, deformation theory, and moduli spaces. We also include suggestions for the use of our realizations in some of these areas as well as in related studies, including enriched category theory and the graph associahedra.

- Quotients of the multiplihedron as categorified associahedra.
  Homotopy, Homology and Applications, vol. 10(2), 227256, 2008.

Arxiv: http://arxiv.org/abs/0803.2694
Journal: http://www.intlpress.com/HHA/v10/n2/

Abstract: We describe a new sequence of polytopes which characterize A_∞ maps from a topological monoid to an A_∞ space. Therefore each of these polytopes is a quotient of the corresponding multiplihedron. Our sequence of polytopes is demonstrated not to be combinatorially equivalent to the associahedra, as was previously assumed in both topological and categorical literature. They are given the new collective name composihedra. We point out how these polytopes are used to parameterize compositions in the formulation of the theories of enriched bicategories and pseudomonoids in a monoidal bicategory. We also present a simple algorithm for determining the extremal points in Euclidean space whose convex hull is the n^{th} polytope in the sequence of composihedra, that is, the n^{th} composihedron $\mathcal{CK}(n)$.

- Operads in iterated monoidal categories (with J. Siehler, E. Seth Sowers)
  Journal of Homotopy and Related Structures 2, 1-43, 2007.

Arxiv: http://arxiv.org/abs/math.AT/0702858
Journal: http://emis.library.cornell.edu/journals/JHRS/volumes/2007/volume2-1.htm

Abstract: The structure of a k-fold monoidal category can be seen as a weaker structure than a symmetric or even braided monoidal category. In this paper we show that it is still sufficient to permit a good definition of (n-fold) operads in a k-fold monoidal category which generalizes the definition of operads in a braided category. Furthermore, the inheritance of structure by the category of operads is actually an inheritance of iterated monoidal structure, decremented by at least two iterations. We prove that the category of n-fold operads in a k-fold monoidal category is itself a $(k-n)$-fold monoidal, strict 2-category, and show that n-fold operads are automatically $(n-1)$-fold operads. We also introduce a family of simple examples of k-fold monoidal categories and classify operads in the example categories.

- Classification of braids which give rise to interchange (with F. Humes)
  Algebraic and Geometric Topology 7, 1233-1274, 2007.

Arxiv: http://arxiv.org/abs/math/0512165
Journal: http://www.msp.warwick.ac.uk/agt/2007/07/p048.xhtml

Abstract: It is well known that the existence of a braiding in a monoidal category $\mathcal{V}$ allows many higher structures to be built upon that foundation. These include a monoidal 2-category $\mathcal{V}$-Cat of enriched categories and functors over $\mathcal{V}$, a monoidal bicategory $\mathcal{V}$-Mod of enriched categories and modules, a category of operads in $\mathcal{V}$ and a 2-fold monoidal category structure on $\mathcal{V}$. These all rely on the braiding to provide the existence of an interchange morphism η necessary for either their structure or its properties. We ask, given a braiding on $\mathcal{V}$, what non-equal structures of a given kind from this list exist which are based upon the braiding. For example, what non-equal monoidal structures are available on $\mathcal{V}$-Cat, or what non-equal operad structures are available which base their associative structure on the braiding in $\mathcal{V}$. The basic question is the same as asking what non-equal 2-fold monoidal structures exist on a given braided category. The main results are that the possible 2-fold monoidal structures are classified by a particular set of four strand braids which we completely characterize, and that these 2-fold monoidal categories are divided into two equivalence classes by the relation of 2-fold monoidal equivalence.

- Vertically iterated classical enrichment
  Theory and Applications of Categories 12, 299-325, 2004.

Arxiv: http://arxiv.org/abs/math/0403214
Journal: http://www.tac.mta.ca/tac/index.html#vol12

Abstract: Lyubashenko has described enriched 2-categories as categories enriched over $\mathcal{V}$-Cat, the 2-category of categories enriched over a symmetric monoidal $\mathcal{V}$. This construction is the strict analogue for $\mathcal{V}$-functors in $\mathcal{V}$-Cat of Brian Day's probicategories for $\mathcal{V}$-modules in $\mathcal{V}$-Mod. Here I generalize the strict version to enriched n-categories for k-fold monoidal $\mathcal{V}$. The symmetric case can easily be recovered. This paper proposes a recursive definition of $\mathcal{V}$-n-categories and their morphisms. We show that for $\mathcal{V}$ k-fold monoidal the structure of a $(k-n)$-fold monoidal strict $(n+1)$-category is possessed by $\mathcal{V}$-n-Cat.

- Enrichment over iterated monoidal categories
  Algebraic and Geometric Topology 4, 95-119, 2004.

Arxiv: http://arxiv.org/abs/math/0403152
Journal: http://www.msp.warwick.ac.uk/agt/2004/04/p007.xhtml

Abstract: Joyal and Street note in their paper on braided monoidal categories that the 2-category $\mathcal{V}$-Cat of categories enriched over a braided monoidal category $\mathcal{V}$ is not itself braided in any way that is based upon the braiding of $\mathcal{V}$. What is meant by "based upon" here will be made more clear in the present paper. The exception that they mention is the case in which $\mathcal{V}$ is symmetric, which leads to $\mathcal{V}$-Cat being symmetric as well. The symmetry in $\mathcal{V}$-Cat is based upon the symmetry of $\mathcal{V}$. The motivation behind this paper is in part to describe how these facts relating $\mathcal{V}$ and $\mathcal{V}$-Cat are in turn related to a categorical analogue of topological delooping first mentioned by Baez and Dolan. To do so I need to pass to a more general setting than braided and symmetric categories – in fact the k-fold monoidal categories of Balteanu, Fiedorowicz, Schwänzl and Vogt. It seems that the analogy of loop spaces is a good guide for how to define the concept of enrichment over various types of monoidal objects, including k-fold monoidal categories and their higher dimensional counterparts. The main result is that for $\mathcal{V}$ a k-fold monoidal category, $\mathcal{V}$-Cat becomes a $(k-1)$-fold monoidal 2-category in a canonical way. I indicate how this process may be iterated by enriching over $\mathcal{V}$-Cat, along the way defining the 3-category of categories enriched over $\mathcal{V}$-Cat.

- Hopf structures on the multiplihedra. (with A. Lauve & F. Sottile)
  submitted preprint, 2009.

Arxiv: http://arxiv.org/abs/0911.2057
preprint: http://faculty.tnstate.edu/sforcey/MSym.pdf

Abstract: We investigate algebraic structures that can be placed on vertices of the multiplihedra, a family of polytopes originating in the study of higher categories and homotopy theory. Most compelling among these are two distinct structures of a Hopf module over the Loday-Ronco Hopf algebra.

- Geometric combinatorial algebras: cyclohedron and simplex. (with D. Springfield) *submitted preprint*, 2009.

Arxiv: http://front.math.ucdavis.edu/0908.3111
preprint: http://faculty.tnstate.edu/sforcey/cyclo_alg.pdf

Abstract: In this paper we report on results of our investigation into the algebraic structure supported by the combinatorial geometry of the cyclohedron. Our new graded algebra structures lie between two well known Hopf algebras: the Malvenuto-Reutenauer algebra of permutations and the Loday-Ronco algebra of binary trees. Connecting algebra maps arise from a new generalization of the Tonks projection from the permutohedron to the associahedron, which we discover via the viewpoint of the graph associahedra of Carr and Devadoss. At the same time that viewpoint allows exciting geometrical insights into the multiplicative structure of the algebras involved. Extending the Tonks projection also reveals a new graded algebra structure on the simplices.

- New interchanging products of Young diagrams. (with G. Eyum, J. Siehler) in preparation, 2008.

Intro.: http://faculty.tnstate.edu/sforcey/newproducts.pdf

Abstract: Interesting combinatorial examples of iterated monoidal categories are given by Young diagrams with lexicographic ordering. Young diagrams are presented as a decreasing sequence of non-negative integers in two ways: the sequence which gives the heights of the columns and that which gives lengths of the rows. We let the tensor product $\otimes_1$ be the product that adds the length of rows of two objects (horizontal stacking) and $\otimes_2$ be the product that adds the height of columns of two objects (vertical stacking). These stacking products together give the Young diagrams the structure of a 2-fold monoidal category. Two higher products of Young diagrams which also form a 2-fold monoidal category are denoted $\boxtimes_1$ and $\boxtimes_2$. Respectively we refer to these as vertical and horizontal multiplication. Both $A \boxtimes_1 B$ and $A \boxtimes_2 B$ are described by first packing each box of diagram B with a copy of A. The two choices for multiplication are the two ways of shifting all the resulting boxes to form a new Young diagram (horizontally then vertically and vice versa).

- *Loop Spaces and Higher-Dimensional Iterated Enrichment*
  Ph.D. Dissertation, Virginia Tech, 2004.

VT: http://scholar.lib.vt.edu/theses/available/etd-04232004-160123/unrestricted/thesisnow.pdf

Abstract: There is an ongoing massive effort by many researchers to link category theory and geometry, especially homotopy coherence and categorical coherence. This constitutes just a part of the broad undertaking known as categorification as described by Baez and Dolan. This effort has as a partial goal that of understanding the categories and functors that correspond to loop spaces and their associated topological functors. Progress towards this goal has been advanced greatly by the recent work of Balteanu, Fiedorowicz, Schwänzl, and Vogt who show a direct correspondence between k-fold monoidal categories and k-fold loop spaces through the categorical nerve.

This thesis pursues the hints of a categorical delooping that are suggested when enrichment is iterated. At each stage of successive enrichments, the number of monoidal products seems to decrease and the categorical dimension to increase, both by one. This is mirrored by topology. When we consider the loop space of a topological space, we see that paths (or 1-cells) in the original are now points (or objects) in the derived space. There is also automatically a product structure on the points in the derived space, where multiplication is given by concatenation of loops. Delooping is the inverse functor here, and thus involves shifting objects to the status of 1–cells and decreasing the number of ways to multiply.

Enriching over the category of categories enriched over a monoidal category is defined, for the case of symmetric categories, in the paper on A_∞-categories by Lyubashenko. It seems that it is a good idea to generalize his definition first to the case of an iterated monoidal base category and then to define $\mathcal{V}$-$(n+1)$-categories as categories enriched over $\mathcal{V}$-n-Cat, the $(k-n)$-fold monoidal strict $(n+1)$-category of $\mathcal{V}$-n-categories where $k < n \in \mathbb{N}$. We show that for $\mathcal{V}$ k-fold monoidal the structure of a $(k-n)$-fold monoidal strict $(n+1)$-category is possessed by $\mathcal{V}$-n-Cat.